

**STRESS AND DEFLECTION OF BEAMS**

No.	Load condition	Bending moment	Reaction force and shear stress	Deflection
1		 $M_x = Px$ $M_{max} = Pa$	 $R_A = P$	 $dc = \frac{Pa^3}{3EI}$ $d_{max} = \frac{Pa^3}{3EI} \left(1 + \frac{3b}{2a}\right)$
2		 $M_{max} = M_x = M$	$R_A = 0$	 $dc = \frac{Ma^2}{2EI}$ $d_{max} = \frac{Ma^2}{2EI} \left(1 + \frac{2b}{a}\right)$
3		 $M_x = \frac{Wx^2}{2a}$ $M_{max} = \frac{Wa}{2}$	 $R_A = W$	 $dc = \frac{Wa^3}{8EI}$ $d_{max} = \frac{Wa^3}{8EI} \left(1 + \frac{4b}{3a}\right)$
4		$M_{max} = W \left(a + \frac{b}{2}\right)$	 $R_A = W$	 $d_{max} = \frac{W}{24EI} \times (8a^3 + 18a^2b + 12ab^2 + 3b^3 + 12a^2c + 12abc + 4b^2c)$
5		$M_x = \frac{Wx^2}{3a^2}$ $M_A = \frac{Wa}{3}$	 $R_A = W$	 $dc = \frac{Wa^3}{15EI}$ $d_{max} = \frac{Wa^3}{15EI} \left(1 + \frac{5b}{4a}\right)$
6		$M_{max} = W \left(a + \frac{2b}{3}\right)$	 $R_A = W$	 $d_{max} = \frac{W}{60EI} (20a^3 + 50a^2b + 40ab^2 + 11b^3)$
7		 $M_{max} = \frac{Pl}{4}$	 $R_A = R_B = \frac{P}{2}$	 $d_{max} = \frac{Pl^3}{48EI}$
8		 $M_{max} = \frac{Pab}{l}$	 $R_A = Pb/l$ $R_B = Pa/l$	 $d_{center} = \frac{Pl^3}{48EI} \left[ \frac{3a}{l} - 4 \left(\frac{a}{l}\right)^3 \right]$ <small><math>d_{max}</math> takes place in the range of 0.0774l from the span center.</small>
9		$M_C = \frac{Pa(b+2c)}{l}$ $M_D = \frac{Pc(b+2a)}{l}$	 $R_A = \frac{P(b+2c)}{l}$ $R_B = \frac{P(b+2a)}{l}$	Deflection of span center is calculated by applying formula given in No.8 to each load.
10		$M_{max} = \frac{Pl}{3}$	 $R_A = R_B = P$	 $d_{max} = \frac{23Pl^3}{648EI}$

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11		$M_C = M_E = \frac{Pl}{4}$ $M_D = \frac{5Pl}{12}$	$R_A = R_B = \frac{3P}{2}$	$d_{max} = \frac{53Pl^3}{1296EI}$
12		$M_C = M_E = \frac{3Pl}{8}$ $M_D = \frac{Pl}{2}$	$R_A = R_B = \frac{3P}{2}$	$d_{max} = \frac{19Pl^3}{384EI}$
13		$M_x = \frac{Wx}{2} (l - \frac{x}{l})$ $M_{max} = \frac{Wl}{8}$	$R_A = R_B = \frac{W}{2}$	$d_{max} = \frac{5}{384} \frac{Wl^3}{EI}$
14		$M_{max} = \frac{W}{b} (\frac{x^2 - a^2}{2})$ where $x_1 = a + \frac{Rab}{W}$	$R_A = \frac{W}{l} (b+c)$ $R_B = \frac{W}{l} (b+a)$	where $a=c$ $d_{max} = \frac{W}{384EI} (8l^3 - 4l^2b^2 - b^3)$
15		$M_{max} = \frac{Wa}{4}$	$R_A = R_B = \frac{W}{2}$	$d_{max} = \frac{Wa(3l^2 - 2a^2)}{96EI}$
16		$M_x = \frac{Wx}{3} (l - \frac{x^2}{l^2})$ $M_{max} = 0.128Wl$ where $x_1 = 0.5774l$	$R_A = \frac{W}{3}$ $R_B = \frac{2W}{3}$	$d_{max} = \frac{0.01304 Wl^3}{EI}$ where $x = 0.5193l$
17		$M_x = Wx(\frac{l}{2} - \frac{2x^2}{3l^2})$ $M_{max} = Wl/6$	$R_A = R_B = \frac{W}{2}$	$d_{max} = \frac{Wl^3}{60EI}$
18		$M_{max} = \frac{W}{4} (l - \frac{b}{3})$	$R_A = R_B = \frac{W}{2}$	$d_{max} = \frac{W}{480EI} (8l^3 + 7al^2 - 4a^2l - 4a^3)$
19		$M_{max} = \frac{Wa}{6}$	$R_A = R_B = \frac{W}{2}$	$d_{max} = \frac{Wa}{240EI} (18a^2 + 20ab + 5b^2)$
20		$-M_A = -M_B = M_C = \frac{Pl}{8}$	$R_A = R_B = \frac{P}{2}$	$d_{max} = \frac{Pl^3}{192EI}$

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21		$M_A = -\frac{Pab^2}{l^2}$ $M_B = -\frac{Pa^2b}{l^2}$ $M_C = \frac{2Pa^2b^2}{l^3}$	$R_A = P \left(\frac{b}{l}\right)^2 \left(1 + 2\frac{a}{l}\right)$ $R_B = P \left(\frac{a}{l}\right)^2 \left(1 + 2\frac{b}{l}\right)$	$d_{max} = \frac{2Pa^2b^2}{3EI(3l-2a)^2} \times \frac{l^2}{3l-2a}$
22		$M_A = M_B = -2Pl/9$ $M_C = M_D = Pl/9$	$R_A = R_B = P$	$d_{max} = \frac{5Pl^3}{648EI}$
23		$M_A = M_B = -19Pl/72$ $M_D = 11Pl/72$	$R_A = R_B = 3P/2$	$d_{max} = \frac{41Pl^3}{5184EI}$
24		$M_A = M_B = -5Pl/16$ $M_D = 3Pl/16$	$R_A = R_B = 3P/2$	$d_{max} = \frac{Pl^3}{96EI}$
25		$M_A = M_B = -\frac{Wl}{12}$ $M_C = \frac{Wl}{24}$	$R_A = R_B = W/2$	$d_{max} = \frac{Wl^3}{384EI}$
26		$M_A = -\frac{W}{12l^2b} [e^4(4l-3e) - c^4(4l-3c)]$ $M_B = -\frac{W}{12l^2b} [d^4(4l-3d) - a^4(4l-3a)]$	$R_A = r_A + \frac{M_A - M_B}{l}$ $R_B = r_B + \frac{M_B - M_A}{l}$ <p><i>r indicates the reaction force of the single span.</i></p>	$d_{max} = \frac{W}{384EI} [l^3 + 2l^2a + 4la^2 - 8a^3]$
27		$M_C = -\frac{Wl}{30} \left( \frac{10x^2}{l^2} - \frac{9x}{l} + 2 \right)$ <p>where <math>+M_{max} = Wl/23.3</math>, <math>x = 0.55l</math>  <math>M_A = -Wl/15</math> <math>M_B = -Wl/10</math></p>	$R_A = 0.3W$ $R_B = 0.7W$	$d_{max} = \frac{Wl^3}{382EI}$ <p>where <math>x_1 = 0.525l</math></p>
28		$M_A = M_B = -\frac{5Wl}{48}$ $M_C = Wl/16$	$R_A = R_B = W/2$	$d_{max} = \frac{1.4Wl^3}{384EI}$
29		$M_A = M_B = -\frac{W}{48l} x$ <p>(<math>5l^2 + 4al - 4a^2</math>)</p>	$R_A = R_B = W/2$	$d_{max} = \frac{W}{1920EI} (17l^3 + 8al^2 + 4a^2l - 16a^3)$
30		$M_A = M_B = -\frac{Wa}{12l} (2l-a)$	$R_A = R_B = W/2$	$d_{max} = \frac{W\alpha^2}{480EI} (5l - 4\alpha)$

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